# International Workshop on "Control and Optimization"

November 7, 2018 Katsura Campus, Kyoto University



Organizer:

Yoshio Ebihara (Kyoto University, Japan) Sponsored by:

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## Program

15:00-15:20

9:00-9:20 Registration Opening Address 9:20-9:30 Morning Session Chair: Yoshio Ebihara (Kyoto University, Japan) 9:30-10:15 Hierarchically Decentralized Control with Global and Local Objectives Shinji Hara (Chuo University, Japan) 10:20-11:05 S-variables for the Positivity Check of Matrix Polynomials with Matrix Indeterminates Dimitri Peaucelle (LAAS-CNRS, Toulouse, France) 11:10-11:55 A New DC Method for Nonlinear Conic Optimization with Application to Problems with BMI Constraints Ellen Hidemi Fukuda (Kyoto University, Japan) Lunch 12:00-13:15 at Cafeteria Afternoon Session I Chair: Izumi Masubuchi (Kobe University, Japan) 13:25-14:10 Extensions of Projection and Rescaling Algorithms Masakazu Muramatsu (The University of Electro-Communications, Japan) An Efficient Approach for Pooling Problems by Second-order 14:15-15:00 Cone Programming Relaxations and Rescheduling Methods Makoto Yamashita (Tokyo Institute of Technology, Japan) in collaboration with Masaki Kimizuka (Hitachi, Ltd., Japan) and Sunyoung Kim (Ewha W. University, South Korea) Tea Break

#### Afternoon Session II

Chair: Dimitri Peaucelle (LAAS-CNRS, Toulouse, France)

15:20-16:05 Facial Reduction for  $H_{\infty}$  State Feedback Control

Hayato Waki (Kyushu University, Japan) in collaboration with Noboru Sebe (Kyushu Institute of Technology, Japan) and

Yoshio Ebihara (Kyoto University, Japan)

16:10-16:55 Error Bounds, Facial Reduction and Amenable Cones

Bruno Figueira Lourenço (Tokyo University, Japan)

#### **Dinner Party**

18:00- Restaurant at City Center



Shinji Hara Chuo University, Japan

**Title:** Hierarchically Decentralized Control with Global and Local Objectives

**Abstract:** There are many large-scale systems that can be regarded as hierarchical networked dynamical systems in a variety of fields related to smart cities. One of the ideas to treat those systems properly from the view point of control is "Glocal (Global/Local) Control," which means that the global purpose is achieved by only local actions of measurement and control. The keys for realization of glocal control are (i) hierarchical networked dynamical systems with multiple resolutions in time and space and (ii) compromise of global and local objectives. After the explanation of the background, we first review stability and robust stability analysis for a class of networked dynamical systems called LTI systems with generalized frequency variables. We then focus on how to design hierarchically decentralized control with global/local objectives and show two design methods. One is based on the standard LQR (Linear Quadratic Regulator) method and the other is based on aggregation using shared model sets. Through the talk we verify the effectiveness of the proposed methods by showing simulation and experimental results of applications to multi-wheel electric vehicle control.



Dimitri Peaucelle LAAS-CNRS, Toulouse, France

**Title:** S-variables for the Positivity Check of Matrix Polynomials with Matrix Indeterminates

Abstract: Positivity check of multivariate polynomials has lived a major breakthrough in the past two decades thanks to sum-of-squares relaxations. These relaxations provide hierarchies of convex semi-definite programming formulations with decreasing and, under mild assumptions, vanishing conservatism. With this very general framework most robust stability analysis are in some sense considered as being solved, at least when leaving numerical issues appart. But when looking at the robust control results, sum-of-squares relaxations are not the unique methodology and are usually combined to other techniques such as S-procedure, KYP-lemma, DG-scalings, vertex separators, S-variables to mention just a few. Based on a joint work with Masayuki Sato, the objective of this talk is to discuss the connections between these various techniques when reformulating the problem as the positivity check of matrix valued polynomials with real and complex, scalar and matrix indeterminates.



Ellen Hidemi Fukuda Kyoto University, Japan

**Title:** A New DC Method for Nonlinear Conic Optimization with Application to Problems with BMI Constraints

Abstract: In control theory, many problems can be reformulated as optimization problems with bilinear matrix inequality (BMI) constraints. One approach for solving these problems was proposed by Dinh et al. (2012), and it uses difference of convex (DC) decompositions. Their method is iterative, and a subproblem, with constraints written as DC functions, has to be solved in each iteration. A regularization term is also added into the subproblems' objective functions. In this work, we improve this method, by allowing change of DC decompositions and regularization terms in each iteration. We also extend the results to nonlinear conic optimization problems, which generalizes in particular, nonlinear programming and nonlinear semidefinite programming. Moreover, we present numerical experiments with specific optimization problems, showing that non-fixed decompositions and regularizations can be a good idea in practice.



Masakazu Muramatsu The University of Electro-Communications, Japan

**Title:** Extensions of Projection and Rescaling Algorithms

**Abstract:** In 2012, Chubanov proposed a projection and rescaling algorithm for computing an interior-feasible solution of homogeneous linear system. Since then, several extensions and variants are proposed; at the beginning of this talk, we survey these results. Then we extend the idea of this algorithm in two ways, each of which can deal with conic programming.

The first extension is to symmetric cone programming. Motivated by Kitahara and Tsuchiya's work on the extension to second-order cone programming, we develop a new projection and rescaling algorithm. Peńa and Soheili also proposed a projection and rescaling algorithm to symmetric cone programming, and we discuss the difference.

The second extension is to linear semi-infinite programming. Specifically, we show that the projection and rescaling algorithm using the oracle model proposed by Chubanov can be extended to semi-infinite programming almost as it is. This algorithm has some resemblance with the one proposed by Dadush, Végh, and Zambelli. The major difference is in condition measures the both algorithms rely on. We also present specific oracles in case of semidefinite programming and second-order cone programming, and derive complexity results for solving them. This is a joint work with Kitahara, Lourenço, Okuno and Tsuchiya.



Makoto Yamashita Tokyo Institute of Technology, Japan

**Title:** An Efficient Approach for Pooling Problems by Second-order Cone Programming Relaxations and Rescheduling Methods

The pooling problem is a mathematical optimization problem Abstract: to determine gas flow in pipeline transportation. Since this problem is a class of network optimization with quadratic constraints and it involves binary variables due to physical conditions, this problem can be described as a mixed-integer nonlinear programming problem. Nishi proposed a method based on semidefinite programming (SDP) relaxations. SDP is a mathematical optimization problem that optimizes a linear function over an linear matrix inequality. Nishi's approach gave a good approximate solution, but demanded a heavy computation cost. In this talk, we consider second-order cone programming (SOCP) relaxations. Generally speaking, since SOCP is a subset of SDP, the computation time of SOCP relaxations is shorter than that of SDP relaxations, meanwhile the objective value obtained from SOCP relaxations is worse than that of SDP relaxations. However, exploiting a mathematical structure of the pooling problem, we prove that the objective value obtained from SOCP relaxations coincides with that of SDP relaxations. Consequently, we can obtain a good approximate solution in a short time. In addition, we develop rescheduling methods to refine the approximate solution obtained from SOCP relaxations. Using the network structure of the pooling problem, we successively adjust the gas flow in the reverse direction, that is, from the sinks to the sources. From numerical experiments, we verified the performance of SOCP relaxations and effectiveness of the rescheduling methods.

### Hayato Waki Kyushu University, Japan

**Title:** Facial Reduction for  $H_{\infty}$  State Feedback Control

**Abstract:** The lack of strict feasibility in Linear Matrix Inequality (LMI) often causes numerical difficulties for accurate computation. Facial reduction (FR) is a remedy to overcome such difficulties. In general, the direct application of FR, however, is comparable to solving the original problem.

We provide a necessary and sufficient condition for the LMI problem of  $H_{\infty}$  state feedback control and its dual to be not strictly feasible. It is closely related to a property in a given linear time invariant dynamical system. We propose a reduction algorithm to the original dynamical system based on the necessary and sufficient condition. This can be regarded as a partial application of FR. I also talk on the positive definiteness of LMI obtained from  $H_{\infty}$  control problem as a second research subject if the time permitted. This is the joint work with Noboru Sebe (Kyushu Institute of Technology, Japan) and Yoshio Ebihara (Kyoto University, Japan).



Bruno Figueira Lourenço Tokyo University, Japan

**Title:** Error Bounds, Facial Reduction and Amenable Cones

**Abstract:** Suppose that we are given two sets  $C_1$ ,  $C_2$  and a point x. Roughly speaking, error bound results allow us to estimate the distance between x and the intersection  $C_1 \cap C_2$  using the individual distances between x and each of the sets  $C_1$ ,  $C_2$ . In this talk, we will discuss *error bounds for conic linear systems*, which includes the special case of linear matrix inequalities (LMIs).

Usually, in order to obtain error bound results, it is necessary to assume constraint qualifications and/or regularity conditions such as Slater's condition. However, in 2000, Jos Sturm showed how error bounds for LMIs can be obtained without assuming regularity conditions and showed that the quality of the bound depends on the *singularity degree* of the problem. The singularity degree, by its turn, is related to *facial reduction*, which is a general approach for regularizing optimization problems.

After reviewing basic facts on facial reduction and taking Sturm's work as a starting point, we show that error bounds without constraint qualifications hold for a broad new family of cones called "amenable cones". Similarly, we show that the quality of the bound is also controlled by the singularity degree of the underlying problem. This highlights that facial reduction and error bounds are intrinsically connected, even in a setting more general than LMIs. In particular, we provide a new Hölderian error bound for the doubly nonnegative cone and for symmetric cones, which recovers Sturm's result as a special case.